Exercise 4

Machine Learning I

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|  | 4A-1. |

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| Prerequisites |

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| Leibnitz rule for constants  Additionally, let at least one of the following conditions hold:   1. is measurable and nonnegative 2. is finite   then we can switch the order of integration according to Tonelli/Fubini, respectively.  Note: In our case, at least one of (i), (ii) is nearly always satisfied. Think about why.  Lastly, we need *Theorem 1* concerning uniform convergence from these notes:  http://www.math.ucla.edu/~tao/resource/general/131bh.1.03s/week45.pdf |

Let be the domain of .

Minimizing the loss cumulative loss for all **t** equals minimizing the loss for each separately. Note: Inside the interior integral, is constant. Let :

We can now solve for

According to the Leibnitz rule, this only holds for finite limits . To extend this proof to the infinite domain, we construct the sequence:

Because probabilities sum to one, if tends to infinity, vanishes for most The will not compensate that, as we required to be finite earlier.  
  
Accordingly, converges to uniformly. Due to , this means the functions converge uniformly to , with   
  
Spoken plainly, this means if we have an infinite domain , we can approximate the solution arbitrarily close by increasing .

Only because the are independent. In our situation this is the case, otherwise we would also have to integrate over all possibilities .

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|  | 4A-2. |

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| Auxiliary calculation |

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| The area of a single infinitesimal -dimensional piece of is . This is trivially an -dimensional extension of the two-dimensional case shown below:  .  Additionally, to convert a function from hyperspherical coordinates into cartesian coordinates , we use the following trigonometric conversion:  i.e.    Lastly, let  Note: describe points on the unit -sphere, so it is no surprise that for all because the radius of the unit sphere is 1.  The centered sphere is described by |

Armed with this knowledge, becomes:

Ergo .

Now we are looking for the maximum density :

Because radii are non-negative, we have a maximum at .  
  
Now if we set we get

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|  | 4A-3. |

Let

Derivative of with respect to :

Conversion to matrix/vector operations:

Generalized for all

Setting zero and solving for :

As usual, denotes the Penrose pseudo inverse.  
Because can be an arbitrary normative factor, it is also possible to write:

Pictures from Python:

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